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Evolution of High Mass Stars:

Once the stars start their main sequence, they remain in an approximate steady state for considerable period of time, given by $t_{nucl} \simeq 10^{10} \left(\frac{M}{M_\odot}\right)^{-2.5}$ yr. The post main sequence evolution depends strongly on the mass of the star.

As we have discussed, the low mass and high mass stars are different in many respects. Low mass stars have radiative cores whereas the high mass stars have convective cores. This implies that the elements are fairly well mixed near the center of high mass stars, leading to a fairly unifor distribution in their core. On the other hand, matter in the central regions of low mass stars is closer to degeneracy. Low mass stars have less central temperature and less central density, but degeneracy depends more critically on the temperature

These features affect further evolution of high mass vs low mass stars. We begin with a discussion of high mass stars.

The borderline between high and low mass stars is $\simeq 3 M_\odot$. A star with mass $M \gtrsim 3 M_\odot$ is considered as a high mass star. To be specific, we consider the evolution of a $5 M_\odot$ star.

Initial Stages in Main Sequence:

As Hydrogen burning in the Core goes on, the Core region will mainly Consist of Helium. This increases the mean molecular weight of the Core, and hence lowers the pressure. The Core responds to this decrease by contracting. Contraction releases gravitational potential energy and increases temperature. As a result, the outer shells expand slightly. The increased luminosity and decreased T_e (because of the expansion) make the star move to the right and up in the H-R diagram.

For a $5M_\odot$ star, the main sequence evolution lasts 6.5×10^7 yr.

Shell Burning of Hydrogen:

Once the Hydrogen in the Core is nearly exhausted, the nuclear luminosity vanishes at the Core. This results in vanishing of the temperature gradient. In the absence of a temperature gradient or density gradient (note that the core is fairly uniform due to convection) there will be no pressure to support the Core. Hence there will be a slight contraction of the star. This has three effects,

(1) The luminosity increases.

(2) A density gradient is established and provides pressure.

(3) The temperature of the edge of the Core increases and the Hydrogen in the shell region undergoes fusion.

During (1) both the luminosity and T_e increase, thus the star moves to the left and up in the H-R diagram. This part of the evolution takes 2.2×10^6 yr.

Hydrogen burning in the shell cause the envelope to expand slightly. This results in a decrease in T_e. The Hydrogen shell around the Core thickens as time goes on. This part of the evolution lasts 1.3×10^6 yr.

For high mass stars ($M \gtrsim 3 M_\odot$) that are powered by the CNO-cycle, the Core has to Collapse by a fair bit for the temperature of the shell increase sufficiently. The situation, as we will see, is different for the low mass stars ($M \lesssim 3 M_\odot$) that are powered by the pp-chain. In that case there is a smooth transition from the core burning to the shell burning.

Core Contraction and Envelope Expansion:

The Helium Core is fairly homogeneous and also isothermal after Hydrogen is enhausted. Its equilibrium depends on the ability of an isothermal core with mass $M_{ic} = qM$ to support an envelope of mass $M_{env} = (1-q)M$. Note that q increases in time as Hydrogen shell burning and its conversion to Helium leads to the growth of Core. It turns out that there is a maximum value for q for which the core can support the envelope. This is the so-called Chandrasekhar-Shoenberg limit:

$$\boxed{q_{max} = \left(\frac{M_{ic}}{M}\right)_{CS} \simeq 0.37 \left(\frac{\mu_{env}}{\mu_{ic}}\right)^2} \quad *$$

Let us see a physical derivation of this limit.

The pressure of the envelope at the core-envelope boundary is:

$$P_{env} = \int_0^{P_{env}} dP = -\int_M^{M_{ic}} \frac{G M_r}{4\pi r^4} \, dM_r \simeq -\frac{G}{8\pi \langle r^4 \rangle} \left(M_{ic}^2 - M^2 \right)$$

Taking $\langle r^4 \rangle \sim \frac{R^4}{2}$, we find:

$$P_{env} \simeq \frac{G M^2}{4\pi R^4}$$

At the boundary, $T_{env} = T_{ic}$, and after using the ideal gas law we have:

$$T_{ic} = \frac{P_{env} \, \nu_{env} \, m_u}{k_B \, \rho_{env}} \quad , \quad \rho_{env} \simeq \frac{3M}{4\pi R^3}$$

This results in:

$$R \simeq \frac{1}{3} \frac{G M}{T_{ic}} \frac{\nu_{env} m_u}{k_B}$$

And:

$$\boxed{P_{env} \simeq \frac{81}{4\pi} \frac{1}{G^3 M^2} \left(\frac{k_B T_{ic}}{\nu_{env} m_u} \right)^4}$$

Note that P_{env} depends on the total mass of the star M.

Using the Virial theorem, we have:

$$4\pi R_{ic}^3 P_{ic} - 3U_{ic} = \Omega_{ic} \quad (P_{ic}: \text{Core pressure at the boundary})$$

Where:

$$U_{ic} = \frac{3 M_{ic} k_B T_{ic}}{\nu_{ic} m_u}, \qquad \Omega_{ic} \simeq -\frac{3}{5}\frac{G M_{ic}^2}{R_{ic}}$$

We then find:

$$P_{ic} = \frac{3}{4\pi R_{ic}^3}\left(\frac{M_{ic} k_B T_{ic}}{\nu_{ic} m_u} - \frac{1}{5}\frac{G M_{ic}^2}{R_{ic}}\right)$$

P_{ic} has a maximum at:

$$R_{ic} = \frac{2}{5}\frac{G M_{ic} \nu_{ic} m_u}{k_B T_{ic}}$$

The maximum value is:

$$\boxed{P_{ic,max} = \frac{375}{64\pi}\frac{1}{G^3 M_{ic}^2}\left(\frac{k_B T_{ic}}{\nu_{ic} m_u}\right)^4}$$

The core can only support the envelope only if $P_{ic,max} > P_{env}$.

This is possible if $q < q_{crit}$, where:

$$\boxed{q_{crit} \simeq 0.54 \left(\frac{\mu_{en}}{\mu_{ic}}\right)^2}$$

This is pretty close to the Chandrasekhar-Shoenberg limit in equation *.

Once M_{ic} exceeds the value given by q_{crit}, the core will contract rapidly. This will happen at a fast rate given by the Kelvin-Helmholtz time scale $t_{KH} \sim \frac{GM^2}{RL}$.

This phase of the evolution is rather short and takes 8×10^5 yr.

(One comment is in order. As the collapse goes on, we reach a point where degeneracy pressure becomes large enough. At that point an isothermal core can support the envelope. For low mass stars degeneracy happens early, and the Chandrasekhar-Schoenberg limit becomes irrelevant.)

The rapid core contraction results in envelope expansion.

This can be understood as follows. If virialization occurs over time scales much shorter than t_{KH}, then we must have $U + \Omega = \text{Const.}$ (Conservation of energy) and $2U + \Omega = 0$ (Virial theorem). This requires that U and Ω be constant individually. For $M_{ic} \gg M_{env}$, we have:

$$|\Omega| \approx \frac{G M_{ic}^2}{R_{ic}} + \frac{G M_{ic} M_{env}}{R}$$

This results in:

$$\frac{dR}{dR_{ic}} \approx -\left(\frac{M_{ic}}{M_{env}}\right)\left(\frac{R}{R_{ic}}\right)^2$$

Therefore core contraction ($R_{ic}\downarrow$) results in the expansion of the envelope ($R\uparrow$). The luminosity of the star arises because of the gravitational contraction of the core over t_{KH}, which results in:

$$L \propto \frac{G M_{ic}^2}{R_{ic}} \left(\frac{G M_{ic}}{R_{ic}^3}\right)^{1/2} \propto R_{ic}^{-5/2} \quad , \quad T_e \propto L^{1/4} R^{1/2} \propto R_{ic}^{-5/8} R^{1/2}$$

We then find:

$$\frac{d\ln L}{d\ln T_e} = \frac{20}{5.4\left(\frac{RM_{ic}}{R_{ic}M_{env}}\right)}$$

Numerical simulations show $RM_{ic} \approx 5 R_{ic} M_{env}$, which results in:

$$\boxed{\frac{d\ln L}{d\ln T_e} \approx 0.8}$$

This relation works well for $M \sim (4-8) M_\odot$. The star moves to the right (expansion lowers T_e) and down in the H-R diagram during this part of its evolution. The location of the nuclear burning shell remains stationary during this process. The thin shell is located at the zero of the velocity field, with matter at the lower radius contracting and matter at the larger radius expanding.

Because the contraction of the core is very rapid, it is difficult to observe stars in this phase of their evolution.

This region of the H-R diagram (for the high mass stars) is thus known as the Hertzsprung gap.

Red Giant Phase and Core Helium Ignition:

With the expansion of the envelope and decrease in T_e, opacity increases (for example, because of the contribution from H^- ions). As a result, a convection zone develops near the surface. The structure of this star is well approximated by the Hayashi Line in this case ($T_e \simeq$ Const., L increasing due to envelope expansion). The convective layers penetrate fairly deeply into the star, hence some of the products of the nuclear reaction can be scooped up and distributed all the way to the stellar surface.

The core contraction leads to an increase in temperature, and the core remains to be non-degenerate. Helium burning can be ignited once the temperature at the core exceeds

$\sim 10^8$ K. Initially, the Helium burning causes the core to expand slightly with the envelope contracting (which is the reverse process of what happens during the rapid core contraction). With the burning of the Helium in the core, the star reaches hydrostatic equilibrium again.

This phase takes $\sim 2\times10^6$ yr for a $5 M_\odot$ star.

Horizontal and Asymptotic Giant Branches:

Once Helium ignition started, the evolution is similar to the preceding main sequence phase (with Hydrogen burning). The Helium burning increases both the luminosity and surface temperature. The star thus moves to the left and up in the H-R diagram.

The Helium burning lasts only $\sim 10^7$ yr, and during both He burning in the core and H burning in the shell contribute to the luminosity.

The change in the mean molecular weight in $3\alpha \to {}^{12}C$ is not as significant as that in $4p \to \alpha$, and hence the slope of trajectory in this phase is different from that in H burning.

As temperature increases, the convective envelope retreats back to the surface, while a convective core develops due to high temperature sensitivity of the $3\alpha \to {}^{12}C$ reaction. The Helium at the core is eventually exhausted, at which point we will have a ${}^{12}C$ core and He burning in the shell. The phase during which core Helium burning takes place is called the horizontal branch. The stars in this region are observed in the H-R diagram because of the slower time scale associated with nuclear burning. The stars in this regime can also develop instabilities in their uter envelope that lead to observable pulsations.

The star undergoes another rapid core contraction (this time a ^{12}C core) and ends up evolving another Hayashi line. This phase is referred to as the asymptotic giant branch.

The evolution up to this point is qualitatively similar for all high mass stars in which the core is non-degenerate when Helium burning is ignited. The number of left-right excursions in the H-R diagram and some other details will depend on the mass of the star. The details of further evolution strongly depend on the mass, which we will discuss in more detail later on.